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Ghost condensates and dynamical breaking of $SL(2, R)$ in Yang–Mills in the maximal Abelian gauge

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Abstract

Ghost condensates of dimension 2 in $SU(N)$ Yang–Mills theory quantized in the maximal Abelian gauge are discussed. These condensates turn out to be related to the dynamical breaking of the $SL(2, R)$ symmetry present in this gauge.

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1. Introduction

Nowadays a great deal of effort is being undertaken to study condensates of dimension two in order to improve our knowledge about the dynamics of Yang–Mills theories in the infrared regime. For instance, the gauge condensate $\langle A^2 \rangle$ has been argued to be suitable for detecting the presence of topological structures such as monopoles [1]. An indication that the vacuum of pure Yang–Mills theory favours a nonvanishing value of this condensate has been achieved in [2] by an explicit two-loop computation of the effective potential in the Landau gauge. A discussion of $\langle A^2 \rangle$ in the context of the operator product expansion and its relevance for lattice QCD may be found in [3]. Further investigations using a recently proposed decomposition [4] of the gauge field have been reported [5].

An interesting mechanism providing a condensate of dimension two has also been proposed [6–8] in the maximal Abelian gauge (MAG). This gauge, introduced by [9, 10], has given evidence for monopole condensation as well as for the Abelian dominance hypothesis, which are the key ingredients for the so-called dual superconductivity [11, 9] mechanism of QCD confinement. An important point to be noted here is that the MAG condition is nonlinear. As a consequence, a quartic ghost interaction term must be necessarily included for renormalizability [12, 13]. As in the case of an attractive four-fermion interaction [14], this term gives rise to an effective potential resulting in a gap equation whose nontrivial solution at weak coupling yields a nonvanishing off-diagonal ghost–antighost condensate $\langle \bar{c}c \rangle$ of dimension two. The physical relevance of this ghost condensate lies in the fact that it is believed to be part of a more general two-dimensional condensate, namely $\left(\frac{1}{2}\langle A^a_\mu A^a_\mu \rangle - \xi \langle \bar{c}^a c^a \rangle\right)$, where

ξ denotes the gauge parameter of the MAG and the index a runs over all the off-diagonal generators. This condensate has been introduced due to its BRST invariance [15] and it is expected to provide effective masses for both off-diagonal gauge and ghost fields [15–17].

Besides the computation of the effective potential, the problem of identifying the symmetry which is dynamically broken by the ghost condensation has also begun to be faced [6–8]. In the case of $SU(2)$, the ghost condensation has been interpreted as a breaking of a global $SL(2, R)$ symmetry [6, 7] displayed by Yang–Mills in the MAG. In [8] the one-loop effective potential for the ghost condensation in the case of $SU(3)$ has been computed. Recently, the authors [18] have been able to establish the existence of $SL(2, R)$ in the MAG, for the general case of $SU(N)$.

The aim of the present work is to continue the investigation on the ghost condensation and their relationship with the dynamical symmetry breaking of $SL(2, R)$, for the general case of $SU(N)$. As already observed [18], the breaking of $SL(2, R)$ can actually occur in different channels, according to which generators are broken. More specifically, the three generators of $SL(2, R)$, namely δ, $\bar{\delta}$ and δ_{FP}, are known [19] to obey the algebra $[\delta, \bar{\delta}] = \delta_{FP}$, where δ_{FP} denotes the ghost number.

The condensate $\langle \bar{c}c \rangle$ analysed in [6–8] corresponds to the breaking of the generators δ, $\bar{\delta}$. In this paper we shall analyse the other off-diagonal condensates $\langle cc \rangle$ and $\langle \bar{c}\bar{c} \rangle$ which are related to the breaking of (δ, δ_{FP}) and $(\bar{\delta}, \delta_{FP})$, respectively [18]. We also remark that the existence of different channels for the ghost condensation has an analogy in superconductivity, known as the BCS[1] versus the Overhauser[2] effect [20]. In the present case the Faddeev–Popov charged condensates $\langle cc \rangle$ and $\langle \bar{c}\bar{c} \rangle$ would correspond to the BCS channel, while $\langle \bar{c}c \rangle$ corresponds to the Overhauser channel. Therefore, the possibility of describing the ghost condensation in the case of $SU(N)$ as a dynamical symmetry breaking of the ghost number seems to be rather natural.

The paper is organized as follows. In section 2 a brief review of the quantization of $SU(N)$ Yang–Mills in the MAG is provided. In section 3 the dynamical symmetry breaking of the ghost number in the case of $SU(2)$ is discussed in detail. Section 4 is devoted to the generalization to $SU(N)$, analysing, in particular, the case of $SU(3)$. In the last section the conclusions are presented.

2. Yang–Mills theory in the MAG

Let $\mathcal{A}_\mu$ be the Lie algebra valued connection for the gauge group $SU(N)$, whose generators T^A, $[T^A, T^B] = f^{ABC}T^C$, are chosen to be anti-Hermitian and to obey the orthonormality condition $\mathrm{Tr}(T^A T^B) = \delta^{AB}$, with $A, B, C = 1, \ldots, (N^2 - 1)$. Following [9, 10], we decompose the gauge field into its off-diagonal and diagonal parts, namely

$$\mathcal{A}_\mu = \mathcal{A}_\mu^A T^A = A_\mu^a T^a + A_\mu^i T^i \tag{2.1}$$

where the index i labels the $N - 1$ generators T^i of the Cartan subalgebra. The remaining $N(N - 1)$ off-diagonal generators T^a will be labelled by the index a. Accordingly, the field strength decomposes as

$$\mathcal{F}_{\mu\nu} = \mathcal{F}_{\mu\nu}^A T^A = F_{\mu\nu}^a T^a + F_{\mu\nu}^i T^i \tag{2.2}$$

with the off-diagonal and diagonal parts given respectively by

$$\begin{aligned} F_{\mu\nu}^a &= D_\mu^{ab} A_\nu^b - D_\nu^{ab} A_\mu^b + g f^{abc} A_\mu^b A_\nu^c \\ F_{\mu\nu}^i &= \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g f^{abi} A_\mu^a A_\nu^b \end{aligned} \tag{2.3}$$

[1] Particle–particle and hole–hole pairing.
[2] Particle–hole pairing.

where the covariant derivative D_μ^{ab} is defined with respect to the diagonal components A_μ^i

$$D_\mu^{ab} \equiv \partial_\mu \delta^{ab} - g f^{abi} A_\mu^i. \tag{2.4}$$

For the Yang–Mills action one obtains

$$S_{\rm YM} = -\frac{1}{4}\int {\rm d}^4x\, \left(F_{\mu\nu}^a F^{a\mu\nu} + F_{\mu\nu}^i F^{i\mu\nu}\right). \tag{2.5}$$

The so-called MAG gauge condition [9, 10] amounts to fixing the value of the covariant derivative $\left(D_\mu^{ab} A^{b\mu}\right)$ of the off-diagonal components. However, this condition being nonlinear, a quartic ghost self-interaction term is required. Following [6–8], the corresponding gauge fixing term turns out to be

$$S_{\rm MAG} = s\int {\rm d}^4x \left(\bar{c}^a\left(D_\mu^{ab}A^{b\mu} + \frac{\xi}{2}b^a\right) - \frac{\xi}{2} g f^{abi}\bar{c}^a\bar{c}^b c^i - \frac{\xi}{4} g f^{abc} c^a \bar{c}^b \bar{c}^c\right) \tag{2.6}$$

where ξ is the gauge parameter and s denotes the nilpotent BRST operator acting as

$$\begin{aligned}
sA_\mu^a &= -\left(D_\mu^{ab}c^b + g f^{abc}A_\mu^b c^c + g f^{abi}A_\mu^b c^i\right) & sA_\mu^i &= -\left(\partial_\mu c^i + g f^{iab}A_\mu^a c^b\right)\\
sc^a &= g f^{abi}c^b c^i + \frac{g}{2} f^{abc}c^b c^c & sc^i &= \frac{g}{2} f^{iab}c^a c^b\\
s\bar{c}^a &= b^a & s\bar{c}^i &= b^i\\
sb^a &= 0 & sb^i &= 0.
\end{aligned} \tag{2.7}$$

Here c^a, c^i are the off-diagonal and the diagonal components of the Faddeev–Popov ghost field, while $\bar{c}^a$, b^a are the off-diagonal antighost and Lagrange multiplier. We also observe that the BRST transformations (2.7) have been obtained by their standard form upon projection on the off-diagonal and diagonal components of the fields. Concerning the gauge parameters, we remark that, in general, the MAG condition allows for the introduction of two independent parameters [12], while in equation (2.6) a unique gauge parameter ξ has been introduced. However, the resulting theory turns out to be renormalizable due to the existence of a further Ward identity which ensures the stability under radiative corrections [13]. Expression (2.6) is easily worked out and yields

$$\begin{aligned}
S_{\rm MAG} = \int {\rm d}^4x \bigg(& b^a\left(D_\mu^{ab}A^{b\mu} + \frac{\xi}{2}b^a\right) + \bar{c}^a D_\mu^{ab} D^{\mu bc} c^c + g\bar{c}^a f^{abi}\left(D_\mu^{bc}A^{c\mu}\right)c^i\\
&+ g\bar{c}^a D_\mu^{ab}\left(f^{bcd}A^{c\mu}c^d\right) - g^2 f^{abi} f^{cdi}\bar{c}^a c^d A_\mu^b A^{c\mu} - \xi g f^{abi} b^a \bar{c}^b c^i\\
&- \frac{\xi}{2} g f^{abc} b^a \bar{c}^b c^c - \frac{\xi}{4} g^2 f^{abi} f^{cdi} \bar{c}^a \bar{c}^b c^c c^d - \frac{\xi}{4} g^2 f^{abc} f^{adi} \bar{c}^b \bar{c}^c c^d c^i\\
&- \frac{\xi}{8} g^2 f^{abc} f^{ade} \bar{c}^b \bar{c}^c c^d c^e\bigg).
\end{aligned} \tag{2.8}$$

Note also that for positive values of ξ the quartic ghost interaction is attractive. As has been shown in [6–8], the formation of ghost condensates at weak coupling is thus favoured.

The MAG condition allows for a residual local $U(1)^{N-1}$ invariance with respect to the diagonal subgroup, which has to be fixed by means of a suitable further gauge condition on the diagonal components A_μ^i of the gauge field. Adopting a covariant Landau condition, the remaining gauge fixing term is given by

$$S_{\rm diag} = s\int {\rm d}^4x\, \bar{c}^i \partial_\mu A^{i\mu} = \int {\rm d}^4x\, \left(b^i \partial_\mu A^{i\mu} + \bar{c}^i \partial^\mu\left(\partial_\mu c^i + g f^{iab} A_\mu^a c^b\right)\right) \tag{2.9}$$

where $\bar{c}^i$, b^i are the diagonal antighost and Lagrange multiplier. As in the familiar case of QED, the diagonal gauge fixing gives rise to a linearly broken $U(1)^{N-1}$ Ward identity [13] which takes the form

$$\mathcal{W}^i S = -\partial^2 b^i \qquad \mathcal{W}^i = \partial_\mu \frac{\delta}{\delta A^i_\mu} + g f^{abi} \left(A^a_\mu \frac{\delta}{\delta A^b_\mu} + c^a \frac{\delta}{\delta c^b} + b^a \frac{\delta}{\delta b^b} + \bar{c}^a \frac{\delta}{\delta \bar{c}^b} \right) \tag{2.10}$$

where $S = S_{\mathrm{YM}} + S_{\mathrm{MAG}} + S_{\mathrm{diag}}$. From (2.10) one sees that the diagonal components A^i_μ of the gauge field play the role of massless photons, while all off-diagonal components behave as charged matter fields.

3. Dynamical ghost number symmetry breaking: the case of $SU(2)$

In this section we discuss the dynamical mechanism which, due to the quartic ghost interaction term, leads to the existence of the off-diagonal condensates $\langle cc \rangle$ and $\langle \bar{c}\bar{c} \rangle$. These condensates will realize a dynamical breaking of the ghost number symmetry. In the case of $SU(2)$ the gauge fixing term (2.8) simplifies to

$$S_{\mathrm{MAG}} = \int \mathrm{d}^4 x \left(b^a \left(D^{ab}_\mu A^{b\mu} + \frac{\xi}{2} b^a \right) + \bar{c}^a D^{ab}_\mu D^{\mu bc} c^c + g \bar{c}^a \varepsilon^{ab} \left(D^{bc}_\mu A^{c\mu} \right) c \right.$$
$$\left. - g^2 \varepsilon^{ab} \varepsilon^{cd} \bar{c}^a c^d A^b_\mu A^{c\mu} - \xi g \varepsilon^{ab} b^a \bar{c}^b c - \frac{\xi}{4} g^2 \varepsilon^{ab} \varepsilon^{cd} \bar{c}^a \bar{c}^b c^c c^d \right) \tag{3.11}$$

where $\varepsilon^{ab} = \varepsilon^{ab3}$ $(a, b = 1, 2)$ are the off-diagonal components of the $SU(2)$ structure constants ε^{ABC}, $c = c^3$ is the diagonal ghost field, and $D^{ab}_\mu = (\partial_\mu \delta^{ab} - g \varepsilon^{ab} A_\mu)$ is the covariant derivative, with $A_\mu = A^3_\mu$ denoting the diagonal component of the gauge connection. In order to deal with the quartic ghost interaction we linearize it by introducing a pair of real[3] auxiliary Hubbard–Stratonovich fields $(\varphi, \bar{\varphi})$, so that

$$-\frac{\xi}{4} g^2 \varepsilon^{ab} \varepsilon^{cd} \bar{c}^a \bar{c}^b c^c c^d \longrightarrow -\frac{1}{\xi g^2} \bar{\varphi}\varphi + \frac{1}{2} \varphi \varepsilon^{ab} \bar{c}^a \bar{c}^b - \frac{1}{2} \bar{\varphi} \varepsilon^{ab} c^a c^b. \tag{3.12}$$

The invariance of the gauge fixed action S under the BRST transformation is guaranteed by demanding that

$$s\bar{\varphi} = \xi g^2 \varepsilon^{ab} b^a \bar{c}^b \qquad s\varphi = 0. \tag{3.13}$$

From expression (3.12) one sees that the requirement of positivity of the gauge fixing parameter, i.e. $\xi > 0$, will ensure that the effective potential $V_{\mathrm{eff}}(\varphi, \bar{\varphi})$ for the Hubbard–Stratonovich fields $(\varphi, \bar{\varphi})$ will be bounded from below, a necessary physical requirement. Moreover, in the following we shall see that a nontrivial vacuum configuration, corresponding to a nonvanishing ghost condensation, will be obtained by setting $\xi = 22/3$.

According to our present aim, the auxiliary fields $(\varphi, \bar{\varphi})$ carry a nonvanishing Faddeev–Popov charge, as can be seen from table 1 where the dimension and the ghost number of all fields are displayed.

Therefore, a nonvanishing vacuum expectation value for $(\varphi, \bar{\varphi})$ will have the meaning of a breaking of the ghost number generator of $SL(2, R)$. In order to analyse whether a nontrivial vacuum for $(\varphi, \bar{\varphi})$ is selected, we follow the Coleman–Weinberg procedure [22] and evaluate the one-loop effective potential $V_{\mathrm{eff}}(\varphi, \bar{\varphi})$ for constant configurations of $(\varphi, \bar{\varphi})$. A straightforward computation gives

$$V_{\mathrm{eff}}(\varphi, \bar{\varphi}) = \frac{\bar{\varphi}\varphi}{\xi g^2} + \mathrm{i} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \ln((-k^2)^2 + \bar{\varphi}\varphi). \tag{3.14}$$

[3] This property follows from the Hermiticity properties of the ghost and antighost fields, chosen here as in [21].

Table 1. Ghost number and canonical dimension of the fields.

Field	A^a_μ	A_μ	c^a	c	$\bar{c}^a$	b^a	φ	$\bar{\varphi}$
Ghost number	0	0	1	1	−1	0	2	−2
Dimension	1	1	1	1	1	2	2	2

Using the dimensional regularization and adopting the renormalization condition at arbitrary scale M

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial\bar{\varphi}\partial\varphi}\right|_{\bar{\varphi}\varphi=M^4} = \frac{1}{\xi g^2} \tag{3.15}$$

the renormalized effective potential is found to be

$$V_{\rm eff}(\varphi,\bar{\varphi}) = \bar{\varphi}\varphi\left(\frac{1}{\xi g^2} + \frac{1}{32\pi^2}\left(\ln\frac{\bar{\varphi}\varphi}{M^4} - 2\right)\right). \tag{3.16}$$

The minimization of $V_{\rm eff}$ yields the condition

$$\ln\frac{\bar{\varphi}\varphi}{M^4} = 1 - \frac{32\pi^2}{\xi g^2} \tag{3.17}$$

which gives the nontrivial vacuum configuration

$$\bar{\varphi} \equiv \bar{v} = \bar{\beta}M^2 \exp\left(\frac{1}{2} - \frac{16\pi^2}{\xi g^2}\right) \qquad \varphi \equiv v = \beta M^2 \exp\left(\frac{1}{2} - \frac{16\pi^2}{\xi g^2}\right) \tag{3.18}$$

where β and $\bar{\beta}$ are dimensionless constants with ghost number $(2, -2)$, obeying the constraint $\beta\bar{\beta} = 1$. Of course, their introduction accounts for $(\varphi, \bar{\varphi})$ being Faddeev–Popov charged. From equation (3.12) one sees that the nonvanishing expectation value of $(\varphi, \bar{\varphi})$ leads to the existence of the ghost condensates $\langle\varepsilon^{ab}c^a c^b\rangle$ and $\langle\varepsilon^{ab}\bar{c}^a\bar{c}^b\rangle$.

In order to analyse the consequences following from the nontrivial ground state configuration (3.18) let us look at the ghost propagators in the condensed vacuum. They are easily computed and read

$$\begin{aligned} &\langle c^a(p)c^b(-p)\rangle = \mathrm{i}\frac{v\varepsilon^{ab}}{(p^2)^2+\bar{v}v} \qquad \langle \bar{c}^a(p)\bar{c}^b(-p)\rangle = -\mathrm{i}\frac{\bar{v}\varepsilon^{ab}}{(p^2)^2+\bar{v}v} \\ &\langle \bar{c}^a(p)c^b(-p)\rangle = \mathrm{i}\frac{p^2\delta^{ab}}{(p^2)^2+\bar{v}v}. \end{aligned} \tag{3.19}$$

One sees thus that, due to the existence of the condensates $\langle\varepsilon^{ab}c^a c^b\rangle$ and $\langle\varepsilon^{ab}\bar{c}^a\bar{c}^b\rangle$, the propagators (3.19) become regular in the low energy region, the infrared cutoff being given by $\bar{v}v$. Moreover, from expressions (3.19) , it follows that another condensate $\langle\bar{c}^a c^a\rangle$ of ghost number zero is nonvanishing, namely

$$\langle\bar{c}^a c^a\rangle = -\frac{(\bar{v}v)^{1/2}}{16\pi}. \tag{3.20}$$

We remark the absence in the propagator $\langle\bar{c}^a(p)c^b(-p)\rangle$ of a term containing the antisymmetric tensor ε^{ab}, forbidding the presence of the condensate $\langle\varepsilon^{ab}c^a\bar{c}^b\rangle$. Note also that all ghost condensates $\langle\varepsilon^{ab}c^a c^b\rangle$, $\langle\varepsilon^{ab}\bar{c}^a\bar{c}^b\rangle$ and $\langle\bar{c}^a c^a\rangle$ are invariant under the residual $U(1)$ transformations, meaning that the corresponding Ward identity (2.10) remains unbroken.

Let us now turn to analyse the stability within the perturbative framework of the vacuum solution (3.18). Consistency with the one-loop computation requires that the vacuum configuration (3.18) is a solution of the gap equation (3.17) at arbitrary small coupling g, ensuring that the logarithmic contributions for the effective potential are small and therefore compatible with the perturbative expansion [22]. This condition will fix the order of magnitude of $(\bar{v}v)$. In order to solve equation (3.17) at small coupling we introduce the renormalization group invariant QCD scale parameter Λ_{QCD},

$$\Lambda_{\mathrm{QCD}}^2 = M^2 \exp\left(-\frac{16\pi^2}{\beta_0 g^2}\right) \tag{3.21}$$

where β_0 is the one-loop coefficient of the β-function of pure Yang–Mills

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} + O(g^5) \qquad \beta_0 = \frac{11}{3}N \qquad \text{for} \quad SU(N). \tag{3.22}$$

Inserting (3.21) in the gap equation (3.17) one gets

$$\ln \frac{\bar{\varphi}\varphi}{\Lambda_{\mathrm{QCD}}^4} = \frac{32\pi^2}{g^2}\left(\frac{1}{\beta_0} - \frac{1}{\xi}\right) + 1. \tag{3.23}$$

Therefore, according to [6–8], the existence of a solution at arbitrary small coupling is ensured by choosing for the gauge parameter ξ, the value

$$\xi = \beta_0 = \tfrac{22}{3}. \tag{3.24}$$

It is worth mentioning here that, as shown in [6–8], the one-loop anomalous dimensions $\gamma_\varphi, \gamma_{\bar{\varphi}}$ of the auxiliary fields $\bar{\varphi}, \varphi$ turn out to vanish when the value of ξ is precisely that of equation (3.24). In turn, this ensures that the one-loop effective potential (3.16) obeys the renormalization group equations. Also, as a consequence of equation (3.24), the breaking $(\bar{v}v)^{1/2}$ turns out to be of the order of Λ_{QCD}^2. Concerning the symmetry breaking aspects related to the existence of the ghost condensates $\langle \varepsilon^{ab} c^a c^b \rangle$, $\langle \varepsilon^{ab} \bar{c}^a \bar{c}^b \rangle$, it is apparent that a nonvanishing expectation value for the Faddeev–Popov charged auxiliary fields φ and $\bar{\varphi}$ leads to a breaking of the ghost number. Let us proceed now with the generalization to the case of $SU(N)$.

4. Generalization to $SU(N)$

In order to generalize the previous mechanism to $SU(N)$ we introduce a set of real Faddeev–Popov charged auxiliary fields $\varphi, \bar{\varphi}$ in the adjoint representation, namely

$$\bar{\varphi} = \bar{\varphi}^A T^A = \bar{\varphi}^a T^a + \bar{\varphi}^i T^i \qquad \varphi = \varphi^A T^A = \varphi^a T^a + \varphi^i T^i \tag{4.25}$$

where the indices a and i run over the off-diagonal and diagonal generators, respectively. The pure off-diagonal ghost terms of the gauge fixing (2.8) can be rewritten as

$$S_{\mathrm{MAG}}^{\mathrm{off}} = \int \mathrm{d}^4 x \left(\bar{c}^a \partial^2 c^a - \frac{1}{\xi g^2}\varphi^i \bar{\varphi}^i + \frac{1}{2}\varphi^i f^{abi} \bar{c}^a \bar{c}^b - \frac{1}{2}\bar{\varphi}^i f^{abi} c^a c^b \right.$$
$$\left. - \frac{1}{\xi g^2}\bar{\varphi}^a \varphi^a + \frac{1}{2\sqrt{2}}\varphi^a f^{abc} \bar{c}^b \bar{c}^c - \frac{1}{2\sqrt{2}}\bar{\varphi}^a f^{abc} c^b c^c \right). \tag{4.26}$$

With the introduction of the auxiliary fields $\varphi^i, \bar{\varphi}^i, \bar{\varphi}^a, \varphi^a$ the Ward identity (2.10) generalizes to

$$\mathcal{W}^i S = -\partial^2 b^i \tag{4.27}$$

where

$$\mathcal{W}^i = \partial_\mu \frac{\delta}{\delta A_\mu^i} + g f^{abi}\left(A_\mu^a \frac{\delta}{\delta A_\mu^b} + c^a \frac{\delta}{\delta c^b} + b^a \frac{\delta}{\delta b^b} + \bar{c}^a \frac{\delta}{\delta \bar{c}^b} + \varphi^a \frac{\delta}{\delta \varphi^b} + \bar{\varphi}^a \frac{\delta}{\delta \bar{\varphi}^b}\right). \tag{4.28}$$

According to this identity, only the $U(1)^{N-1}$-invariant diagonal fields $\varphi^i, \bar{\varphi}^i$ may acquire a nonvanishing vacuum expectation value. As before, let us look at the one-loop effective potential, which in the present case reads

$$V_{\text{eff}}(\varphi^i, \bar{\varphi}^i) = \frac{\bar{\varphi}^i \varphi^i}{\xi g^2} + \frac{\mathrm{i}}{2} \ln \det \mathcal{M}^{ab} \tag{4.29}$$

where $\mathcal{M}^{ab}$ denotes the $(2N(N-1)) \times (2N(N-1))$ matrix

$$\mathcal{M}^{ab} = \begin{pmatrix} f^{abi}\varphi^i & \delta^{ab}\partial^2 \\ -\delta^{ab}\partial^2 & -f^{abi}\bar{\varphi}^i \end{pmatrix}. \tag{4.30}$$

In the case of $SU(3)$, the Cartan subgroup has dimension 2, with $\varphi^i = (\varphi^3, \varphi^8)$ and $\bar{\varphi}^i = (\bar{\varphi}^3, \bar{\varphi}^8)$. Making use of the explicit values of the structure constants, the effective potential (4.29) is found to be

$$V_{\text{eff}}(\varphi^i, \bar{\varphi}^i) = \frac{\bar{\varphi}^i \varphi^i}{\xi g^2} + \mathrm{i} \sum_{\alpha=1}^{3} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \ln\left((-k^2)^2 + \left(\varepsilon_\alpha^i \bar{\varphi}^i\right)\left(\varepsilon_\alpha^j \varphi^j\right)\right) \tag{4.31}$$

where ε_α are the root vectors of $SU(3)$, given by $\varepsilon_1 = (1, 0), \varepsilon_2 = (-1/2, -\sqrt{3}/2)$ and $\varepsilon_3 = (-1/2, \sqrt{3}/2)$. We observe that expression (4.29), although obtained in a different way, is very similar to that of [8]. The effective potential (4.29) turns out to possess global minima along the directions of the roots, given by the following configurations $\left(\varphi_\alpha^3, \bar{\varphi}_\alpha^3, \varphi_\alpha^8, \bar{\varphi}_\alpha^8\right)$,

$$\begin{aligned} &\varphi_1^3 = 2^{1/3}\beta M^2 \exp\left(\frac{1}{2} - \frac{32\pi^2}{3\xi g^2}\right) \qquad \bar{\varphi}_1^3 = 2^{1/3}\bar{\beta} M^2 \exp\left(\frac{1}{2} - \frac{32\pi^2}{3\xi g^2}\right) \\ &\varphi_1^8 = \bar{\varphi}_1^8 = 0 \end{aligned} \tag{4.32}$$

$$\begin{aligned} &\varphi_2^3 = 4^{-1/3}\beta M^2 \exp\left(\frac{1}{2} - \frac{32\pi^2}{3\xi g^2}\right) \qquad \bar{\varphi}_2^3 = 4^{-1/3}\bar{\beta} M^2 \exp\left(\frac{1}{2} - \frac{32\pi^2}{3\xi g^2}\right) \\ &\varphi_2^8 = \sqrt{3}\varphi_2^3 \qquad \bar{\varphi}_2^8 = \sqrt{3}\bar{\varphi}_2^3 \end{aligned} \tag{4.33}$$

and

$$\begin{aligned} &\varphi_3^3 = 4^{-1/3}\beta M^2 \exp\left(\frac{1}{2} - \frac{32\pi^2}{3\xi g^2}\right) \qquad \bar{\varphi}_3^3 = 4^{-1/3}\bar{\beta} M^2 \exp\left(\frac{1}{2} - \frac{32\pi^2}{3\xi g^2}\right) \\ &\varphi_3^8 = -\sqrt{3}\varphi_3^3 \qquad \bar{\varphi}_3^8 = -\sqrt{3}\bar{\varphi}_3^3 \end{aligned} \tag{4.34}$$

with M^2 being the renormalization scale and $\beta\bar{\beta} = 1$. Expression (4.31) takes the same value for all minima. As in the previous case of $SU(2)$, from the requirement that the nontrivial vacuum configurations (4.32)–(4.34) are solutions of the gap equation at weak coupling, for ξ one obtains the value

$$\xi = \frac{2\beta_0}{3} = \frac{22}{3}. \tag{4.35}$$

It is worth underlining that the value obtained for ξ is precisely the same as in the case of $SU(2)$.

5. Conclusion

The existence of the off-diagonal ghost condensates $\langle cc \rangle$ and $\langle \bar{c}\bar{c} \rangle$ of dimension 2 in the MAG has been discussed. These condensates rely on the nonlinearity of the MAG gauge fixing condition, which requires the introduction of a quartic ghost self-interaction term, needed for the renormalizability of the model. For positive values of the gauge parameter ξ the quartic self-interaction is attractive, favouring the formation of ghost condensates, which show up as nontrivial solutions of the gap equation for the effective potential.

A further important point is that the gauge parameter ξ can be chosen to ensure that the condensed vacuum configuration is a solution of the gap equation at weak coupling, i.e. for small values of the gauge coupling constant g. The whole framework is thus consistent with the perturbative loop expansion. In particular, for ξ one obtains the value $22/3$, for both $SU(2)$ and $SU(3)$. Although we cannot extend the validity of this mechanism to the strong coupling region, i.e. to energy scales below Λ_{QCD}, it can be interpreted, to some extent, as possible evidence for the Abelian dominance. It is indeed worth recalling that the off-diagonal condensate $\langle \bar{c}c \rangle$ is part of the more general condensate $\left(\frac{1}{2}\langle A^a_\mu A^{\mu a}\rangle - \xi\langle \bar{c}^a c^a\rangle\right)$, which is expected to provide effective masses for all off-diagonal fields [15–17].

In this work, the ghost condensation has been related to the dynamical breaking of the ghost number symmetry, which is present in Yang–Mills theory with arbitrary gauge group. It is useful to recall that the ghost number generator δ_{FP} is part of $SL(2, R)$, which is present in the MAG for any gauge group $SU(N)$ [18].

Aspects concerning the Goldstone boson associated with this breaking as well as the characterization of the condensate $\left(\frac{1}{2}\langle A^a_\mu A^{\mu a}\rangle - \xi\langle \bar{c}^a c^a\rangle\right)$ are under investigation. We remark that this massless excitation should be identified with a bound state of ghosts. This follows by noting that, classically, the auxiliary fields φ^i and $\bar{\varphi}^i$ correspond to the ghost composite operators $f^{iab}c^a c^b$ and $f^{iab}\bar{c}^a\bar{c}^b$. This massless excitation is expected to decouple from the physical spectrum. In fact, in the case of $SU(2)$, a decoupling argument based on the quartet mechanism [21] has been given for the Goldstone boson related to the breaking of $SL(2, R)$ [6, 7].

Let us conclude with some general comments on the result so far obtained. Certainly, many aspects of the ghost condensation remain to be analysed, deserving a deeper understanding. Until now, the ghost condensates have been investigated at one-loop order and in the weak coupling regime, where a nontrivial vacuum seems to emerge. However, a complete analysis should include a better understanding (at least qualitatively) of the strong coupling. This would require facing genuine nonperturbative effects, such as Gribov's ambiguities, which are also present in the maximal Abelian gauge [23]. Here, the employment of the Schwinger–Dyson equations [24] could provide more information about the role of the ghost condensates for the infrared region of Yang–Mills theories.

Also, a two-loop analysis of the effective potential could improve our understanding of the weak coupling regime and of the relationship (3.24) between the β-function and the gauge parameter ξ . The combined use of the local composite operators technique [2] and the algebraic renormalization [25] proves to be particularly useful for this kind of analysis, as done in the case of the gluon–ghost condensate $\left\langle \frac{1}{2}A^2 - \xi\bar{c}c\right\rangle$ in the covariant nonlinear Curci–Ferrari gauge [26].

It is worth mentioning that by now evidence for the ghost condensation has been reported in other gauges, namely in the Curci–Ferrari gauge [27–29] and in the Landau gauge [30]. All these gauges, including the maximal Abelian gauge, possess a global $SL(2, R)$ symmetry [18], a feature which seems to be deeply related to the ghost condensation.

Other important aspects to be further analysed are those related to the BCS versus Overhauser effect, i.e. to establish which is the preferred vacuum with the lowest energy. Also, the role of the BRST symmetry in the presence of ghost condensation needs to be clarified. We remark that these aspects have been recently investigated in detail in [31] in the case of the Curci–Ferrari and Landau gauge. Here, it turns out that, due to the $SL(2, R)$ invariance of the effective potential, both the BCS and the Overhauser vacua can be consistently chosen as vacuum state. Furthermore, the resulting theory perturbed around the condensed vacuum is found to be BRST invariant. A similar analysis is expected to apply in the maximal Abelian gauge, leading essentially to the same conclusion.

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